Study of Fat-tail Risk

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I. Introduction

During periods of financial inclemency, investors often look to ride out the storms in vehicles that will protect their assets and preserve their net worth. In 2008, there were few, if any, safe harbors, as virtually all equity, debt and alternative investment asset classes suffered under a perfect storm of bad news. Once mighty businesses, such as Lehman Brothers, Merrill Lynch and AIG, who had withstood many economic storms over their long and storied histories, succumbed under the heavy weight of leverage, inadequate capital, poor liquidity and the marketplace's complete lack of risk tolerance.

The Year 2008 is to be remembered as the most challenging year for the hedge fund industry, when a number of hedge fund managers – at least 20% of over 10,000 – are expected to go out of business. By definition, hedge funds take long and short positions in various securities, and "enhance" the performance with leverage. By using leverage, a diminutive arbitrage can yield magnificent returns. Conversely, leverage will multiply the damage from a small mistake: a fact many tend to forget – particularly when the system is awash with liquidity.

Thanks to the rapid development of financial engineering, investors have become better equipped at managing predictable portfolio risks. Unfortunately, throughout financial history, there have been a number of extreme, and often severe, events that cannot be predicted based on prior events. While Nassim Taleb famously referred to this as the Black Swan theory, it is more widely regarded as "Fat-tail Risk". The purpose of this study is to illustrate the existence of such risk and its historical frequency, paying particular attention to recent events.

II. Data

For our analysis of fat-tail risk in the U.S. stock markets, we used the S&P 500 Index's daily returns (the S&P Daily Return Data) from December 30, 1927 through November 21, 2008[1] (Source: Bloomberg). The S&P 500 Index is an asset-weighted stock index commonly used as representative of the U.S. stock markets.

III. Definition

A fat tail is a property of probability distributions exhibiting extremely large kurtosis, particularly relative to the ubiquitous normal distribution which itself is an example of an exceptionally thin tail distribution. In academic terms, the condition of probability distribution that exhibits fat tail(s) is called *leptokurtosis*. A fat-tail risk in financial markets refers to extreme swings in the markets which cannot be predicted solely based on the normal distribution of the return probability. Sigma, or 'σ', is used as a parametric standard deviation of the S&P Daily Return Data. In this example, σ is the average daily deviation from the expected return of the market.

IV. Methodologies

To illustrate the presence of fat-tail risk in the U.S. stock markets, two different methodologies were conducted. The first methodology is to calculate the daily returns of the S&P 500 Index and

[1] The S&P 500 index was created in 1957, but it has been extrapolated back in time. The first S&P index was introduced in 1923. Prior to 1957, the primary S&P stock market index consisted of 90 companies, known as the "S&P 90, and was published on a daily basis. A broader index of 423 companies was also published weekly. In order to capture the movement of broader stock markets than the Dow Jones Industrial Average, which is an index of 30 companies, the S&P index was chosen for the analysis of this paper.

to compute statistics to examine whether the actual distribution of returns exhibits statistical characteristics of leptokurtosis.

The second methodology is to compare the frequencies of distributions falling into certain ranges of daily returns. The distribution range, or the Daily Return Range, is based on the distance from the mean, which is calculated as the number of standard deviation from the mean, or σ.

V. *Statistics*

Table 1 shows the statistics of the S&P Data, including Mean (0.03%), Standard Deviation (1.18%), Kurtosis (18.35) and Skewness (-0.10). The Kurtosis – a measure of peakedness – of a normal distribution is 0 with a kurtosis of 18.21 indicative of a more variable, wider shaped distribution, i.e. the distribution has "fat tails". The skewness is a measure of the asymmetry in the distribution curve with negative skewness indicating that the curve has a longer left (or negative) tail. That is, there are a greater number of extreme negative daily returns than extreme positive daily returns.

Table 1: Descriptive Statistics for S&P Daily Return Data (1927 ~ 2008)

Sample Number	20,319
Mean	0.026%
Standard Deviation	1.182%
Kurtosis	18.347
Skewness	-0.098
Maximum Daily Return	16.61%
Minimum Daily Return	-20.47%

Based on the statistic in Table 1, we classified the S&P Daily Return Data into 13 categories as shown in Table 2. The more +/-6σ we observe, the more evidently Fat-tail Risk exists in the S&P

Daily Return. We also define an observation whose Daily Return Range exceeds 4 standard deviations from the mean as a Fat-tail Day.

Table2: S&P Daily Return Data – Standard Deviation Ranges

# of Standard Deviations from Mean	Daily Return Range
+6σ	Above +7.05%
+5σ	+5.88% ~ +7.05%
+4σ	+4.71% ~ +5.88%
+3σ	+3.53% ~ +4.71%
+2σ	+2.36% ~ +3.53%
+1σ	+1.19% ~ +2.36%
0σ	-1.14% ~ +1.19%
-1σ	-2.31% ~ -1.14%
-2σ	-3.48% ~ -2.31%
-3σ	-4.65% ~ -3.48%
-4σ	-5.82% ~ -4.65%
-5σ	-7.00% ~ -5.82%
-6σ	Below -7.00%

VI. Observations

Under the statistical normal distribution of performance returns, deviations from the mean return should occur with a certain frequency; the larger the deviance, the lower the frequency. Table 3 shows that for the daily performance of the S&P 500, the normal distribution significantly underestimates the probability of having days with very significant negative returns, which we define as being four or more standard deviations from the mean. As an example, whereas the normal distribution of the daily return of the S&P would suggest a negative three-sigma event (-3.5% daily return) should have occurred 27 times over the last one hundred years, this has actually occurred 100 times in the 81 years since 1927. When one looks at even greater negative return days, the results become even more pronounced. As one will see from the chart below, the "normal" likelihood of a negative four-sigma event (-4.7% daily return) is once in one hundred years; yet we have seen this take place 43 times since 1927. The same normal distribution

suggests virtually no possibility (.00003%) of a day where negative returns are greater than 5.8%, but, once again, we have witnessed such days on 40 occasions in the last 81 years, and alarmingly, three times in 2008 alone.

Table 3: S&P Daily Return Data – Actual vs Normal Distribution

	Actual Distribution		Normal Distribution	
# of Standard Deviations from Mean	Observed	Percentage	Predicted	Percentage
+6σ	26	0.13%	0	0.00%
+5σ	13	0.06%	0	0.00%
+4σ	34	0.17%	1	0.00%
+3σ	89	0.44%	27	0.13%
+2σ	276	1.36%	435	2.14%
+1σ	1,393	6.86%	2,761	13.59%
0σ	16,603	81.71%	13,872	68.27%
-1σ	1,377	6.78%	2,761	13.59%
-2σ	325	1.60%	435	2.14%
-3σ	100	0.49%	27	0.13%
-4σ	43	0.21%	1	0.00%
-5σ	19	0.09%	0	0.00%
-6σ	21	0.10%	0	0.00%
Total	20,319	100%	20,319	100%

Chart 2 and 3 are graphical presentations of the Actual and Normal Distribution of the S&P Daily Return Data. Chart 2 illustrated the peakness of the observed data (green area) relative to the normal distribution. Meanwhile, chart 3 exhibits the relative frequency of extreme, negative events compared to that predicted by the bell curve.

Chart 2: Actual S&P Daily Returns vs Normal Distribution

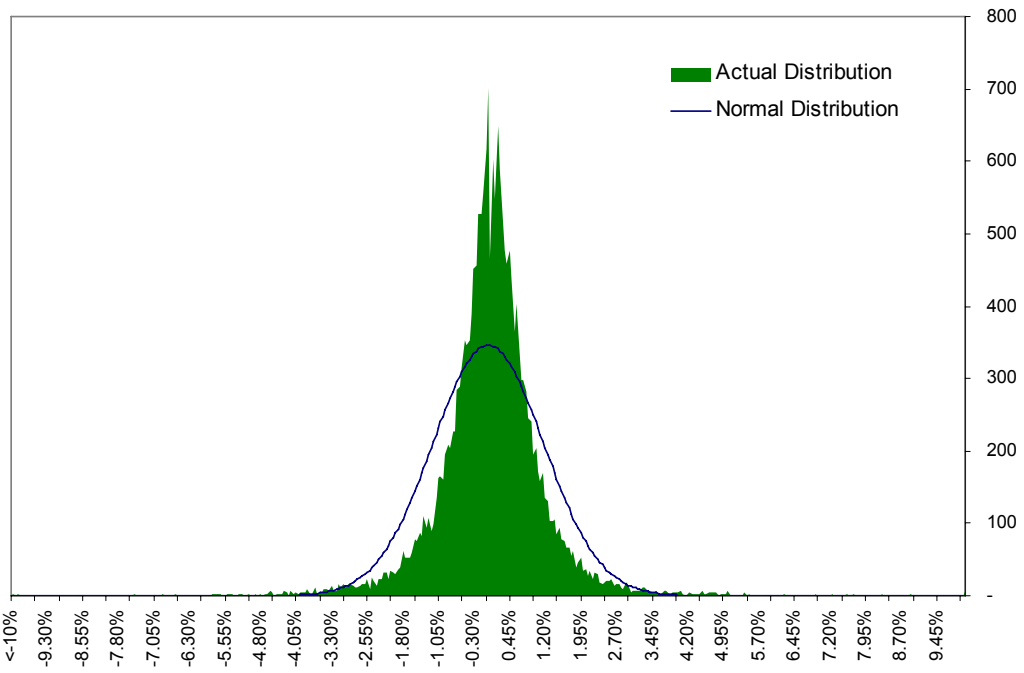
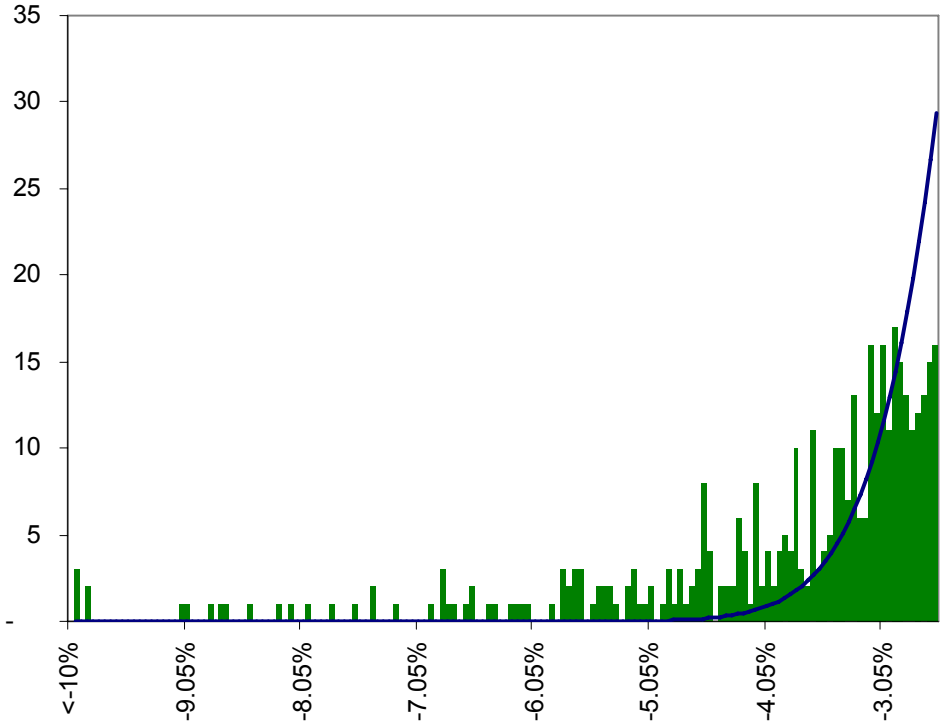


Chart 3: Actual S&P Daily Returns vs Normal Distribution (from -3% to -10%)



The Table 4 shows the historical trend of the Fat-tail risk decade by decade. In addition to the frequency of fat-tail days, one can also observe their uneven distribution over time. During the period of 1930 ~ 1939, when the US economy suffered a severe economic slump, or the Great Depression, there is a higher frequency of Fat-tail events.

Table 4: S&P Daily Return Data - Decade-by-Decade Analysis

		-4σ Daily Return		-5σ Daily Return		-6σ Daily Return	
Decade	Return	Days	%	Days	%	Days	%
1927 ~ 1929	21.46%	4	0.800%	4	0.400%	3	0.600%
1930 ~ 1939	-41.91%	28	1.122%	28	0.280%	11	0.441%
1940 ~ 1949	34.75%	3	0.120%	3	0.040%	2	0.080%
1950 ~ 1959	256.70%	1	0.040%	1	0.040%	0	0.000%
1960 ~ 1969	53.72%	0	0.000%	0	0.040%	0	0.000%
1970 ~ 1979	17.25%	0	0.000%	0	0.000%	0	0.000%
1980 ~ 1989	227.40%	2	0.079%	2	0.079%	2	0.079%
1990 ~ 1999	315.75%	0	0.000%	0	0.079%	0	0.000%
2000 ~ 2008YTD	-45.55%	8	0.358%	3	0.134%	3	0.134%
Total		43	0.212%	19	0.094%	21	0.103%
Normal Distribution			0.003%		0.000%		0.000%

After the 20% crash of stock market on October 19, 1989, best known as Black Monday, the U.S. stock market grew for an entire decade without experiencing any fat-tail days. Even the day after September 11 terrorist attacks, the stock market fell by only 4.9%, a -4σ daily return. This “quiet” market environment changed meaningfully in 2008.

Triggered by the subprime mortgage losses, the global financial system was dealt a blow and financial activity slowed significantly. A number of financial institutions failed, were acquired, or had to be bailed out by the government. The stock market was not an exception. As shown in Table 5, there was a significant increase in the frequency of Fat-tail Days and, as of November 21, 2008, the only year with a higher frequency of such events is 1932, the middle of the Great Depression.

Table 5: S&P Daily Return Data - Selected Year Analysis

		-4σ Daily Return		-5σ Daily Return		-6σ Daily Return	
Selected Year	Return	Days	%	Days	%	Days	%
1932	-14.80%	10	4.00%	2	0.80%	4	1.60%
2008YTD	-45.52%	6	3.00%	3	1.50%	3	1.50%
1929	-28.50%	4	1.61%	2	0.80%	3	1.21%
1931	-47.10%	2	0.79%	0	0.00%	2	0.79%
1987	2.00%	1	0.40%	0	0.00%	2	0.79%
1937	-38.60%	4	1.60%	0	0.00%	1	0.40%
1940	-15.10%	2	0.80%	1	0.40%	1	0.40%
1930	-28.50%	1	0.40%	0	0.00%	1	0.40%
2000	-10.10%	1	0.40%	0	0.00%	0	0.00%
1941	-17.90%	0	0.00%	0	0.00%	0	0.00%
1966	-13.10%	0	0.00%	0	0.00%	0	0.00%
1973	-17.40%	0	0.00%	0	0.00%	0	0.00%
1974	-29.70%	0	0.00%	0	0.00%	0	0.00%
1998	26.70%	0	0.00%	1	0.40%	0	0.00%
2002	-23.40%	0	0.00%	0	0.00%	0	0.00%
All		36		9		17	
Normal Distribution			0.003%		0.000%		0.000%

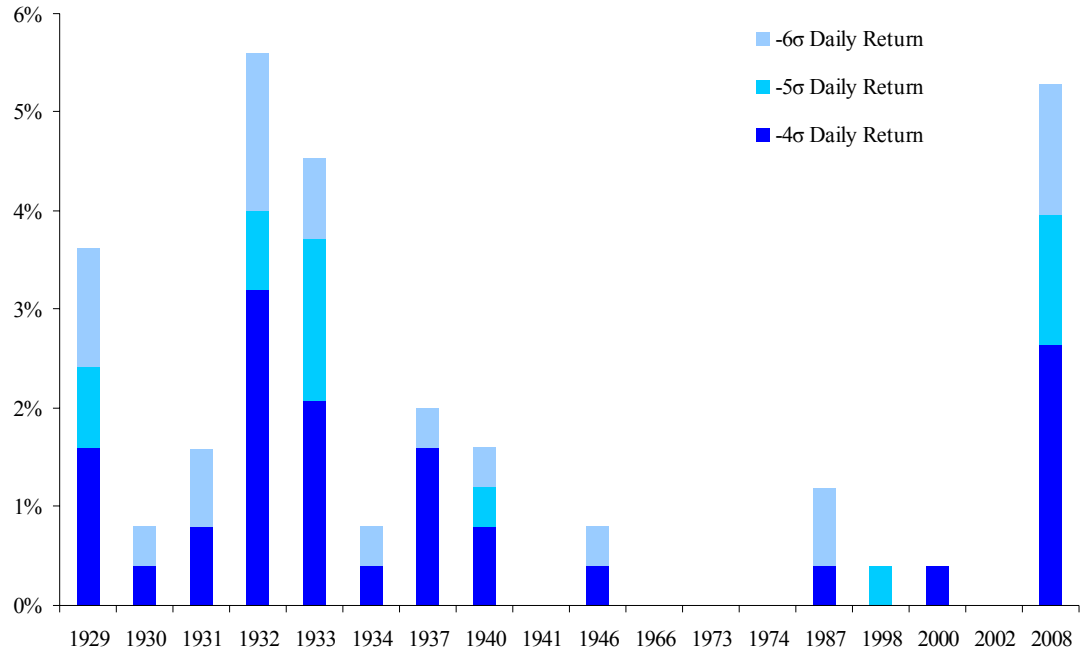
Similarly, Table 6 shows the list of -6σ days since 1927. Prior to 2008, and with the exception of Black Monday, all -6σ Days were observed between 1920 and 1940.

Table 6: -6σ Days in History

Date	Daily Return
10/19/1987	-20.47%
10/28/1929	-12.94%
10/29/1929	-10.16%
11/6/1929	-9.92%
9/3/1946	-9.91%
10/18/1937	-9.12%
10/5/1931	-9.07%
10/15/2008	-9.03%
7/20/1933	-8.88%
9/29/2008	-8.79%
7/21/1933	-8.70%
10/10/1932	-8.55%
10/26/1987	-8.28%
10/5/1932	-8.20%
8/12/1932	-8.02%
7/26/1934	-7.83%
6/16/1930	-7.64%
10/9/2008	-7.62%
5/14/1940	-7.47%
5/31/1932	-7.45%
9/24/1931	-7.29%

Lastly, Chart 4 is a graphical representation of fat-tail days in selected years. As discussed above, a high frequency of fat-tail days was also observed in the 1930s.

Chart 4: Frequency of Extreme Events in Selected Years



VII. Conclusion

The purpose of this paper was to elucidate upon the term *Fat-tail Risk* and examine its existence in the U.S. stock market. When compared to a normal distribution, historical data has shown a significant degree of such risk in the returns of the S&P Daily Return Data. As such, this study has shown that the stock market has experienced a far more volatile trading environment than assumed by a simple bell-curve (normal distribution).

Many investors believed that the Great Depression was a historical anomaly and that the volatility experienced by the stock markets during that period would not occur again, at least not with such frequency. The conditions the market is observing today is, however, as extreme, if not more so than, what was experienced over 70 years ago.

A number of hedge fund strategies designed with the assumption that fat-tail risks are negligible, worked well for years. In 2008, however, such strategies finally revealed their vulnerability to fat-tail risk and subsequently failed. Those investing in hedge funds should be aware of the negative consequences of fat-tail risk and avoid those strategies that overly exposed to such risk.

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